Online Appendix for Costly Verification and Money Burning

Rohit Patel* Can Urgun†

May 2022

Lemma 1. *An optimal mechanism is equivalent to the following plan:*

1. *The principal announces a policy that specifies a selection policy, an inspection policy and a money burning policy*
2. *Agents report their types to the principal*
3. *Following the selection policy, an agent is selected for Stage 4*
4. *The selected agent burns the required money*
5. *The selected agent's claim is inspected according to the inspection policy*
6. *The selected agent receives the object if there is no inspection, or if the agent is found truthful upon inspection*

Proof. The proof is an extension of the argument in Ben-Porath et al. (2014) and follows along similarly in three steps.

Step 1: We can restrict attention to truth-telling equilibria in direct mechanisms Consider an arbitrary mechanism which includes multiple rounds, cheap talk statements, repeated checking and money burning eventually culminating in allocation to an agent or even possibly withholding. Take an equilibrium σ of this convoluted mechanism and construct a mechanism as follows. Each agent i reports a type t_i, given a vector of reports

*Meta
†Princeton University

the principal simulates what would happen in the mechanism, checking and asking agents to burn money as required. If no check reveals a lie and all money burning is done appropriately by the agents then the principal allocates using the simulation. If more than one agent has lied or more than one agent has failed to burn money the principal allocates the good arbitrarily as only unilateral deviations matter. If a single agent is found to be lying but they had no more moves the principal completes the simulation taking the lying into account. On the other hand if a single agent is found out to be lying and the lying agent had any more moves then the principal cannot continue the simulation. Therefore, the principal arbitrarily picks a feasible strategy for the continuation for the lying agent and completes the simulation and allocates the object. If a single agent fails to burn the required money then the principal treats it as a deviation from the equilibrium by the announced type and continues with the simulation, keeping track of multiple deviations if they occur. If the same agent is found to be lying as well in the continuation, again the principal arbitrarily picks a feasible strategy for the continuation for the lying agent completing the simulation and allocating the object. It is easy to see that truth telling is an equilibrium of this game. The reasoning is that any deviation of telling a different type corresponds to a certain strategy in the original mechanism. If all other agents are playing truthfully then it is a best response for each agent to report their type truthfully. Not deviating from the money burning required is also a best response as any different amount of money burning constitutes a detectable deviation in the original mechanism and once again following along the associated money burning was a best reply. Clearly the principal's payoff is the same as in the original mechanism.

Step 2: In an optimal mechanism money burning is done before a claim is checked. First observe that by the fully revealing nature of the checks, there is no need for any agent to burn money after that agent is checked on the equilibrium; their type is fully revealed and money burning is costly to both the agent and the principal. By definition of truthful equilibria all situations where an agents check reveals them to be lying or an agent fails to do the required money burning is off the equilibrium path. If a claim turns out the be false the only punishment tools available are withholding the object or asking the agent to burn money. If the agent is asked to burn money without having a chance to get the object than the agent will also refuse to do so due to individual rationality. If the principal asks the agent to burn money as punishment while still offering a chance to get the object (albeit different) then the principal must also burn the same amount of money, despite knowing the type of the agent perfectly. The principal can just alter the probability of allocation after lying to avoid the money burning cost without changing the incentives of the agent to keep them truthful. Thus there is no point in asking an agent to burn money after their type is revealed even as a punishment. Since the principal cannot punish by money burning then a check revealing the agent has lied must result in that agent receiving allocation with 0 probability. Since each agent's money burning precedes their own check, now we can appeal to a similar construction as in Ben-Porath et al. (2014) to show that all checks can be done simultaneously. For any truthful mechanism (potentially involving multiple rounds of checks across different agents) the principal can calculate the probability that a type t_i is going to be checked given any profile of reports. Then, instead of going through the sequence can just use this probability to randomize over the set of agents to check simultaneously. For example if the principal were to ask agent 1 to burn money first then check their claim, then with some probability ask agent 2 to burn some money and check their claim afterwards, the principal can just randomize between asking both agents to burn money and check their claims or just asking agent 1 to burn money and check agent 1's claim only. With perfect verification it is easy to see that truth telling is an equilibrium of the simultaneous check version of the same game and the payoff to the principal remains unchanged. And by the above argument any money burning required by any type will have to be done prior to this simultaneous check.

Step 3: In an optimal mechanism the principal chooses at most one agent to burn money first then potentially checks their claim. If an agent's report is checked even when he would not receive the object if found to have told the truth, his incentives to report honestly are not affected. Since checking is costly for the principal, this means that if the principal checks an agent, and if the agent is found to have been honest, that agent must receive the object with probability 1. Similarly, if an agent is asked to burn money then that agent must have a positive probability of allocation at that point. Since deviation from money burning is a detectable deviation which is punished by withholding all agents under the truthful equilibria follow the suggested money burning amount. But then, since money burning is costly to the principal the principal only needs to ask one agent to burn money after the initial report of types. □

Lemma 2. *$g(S)$ is strictly submodular, i.e. for any two sets S, S' where $S \not\subset S'$ and $S' \not\subset S$ we have $g(S \cup S') + g(S \cap S') < g(S) + g(S')$.*

Proof. Let $h(S) = 1 - g(S)$, and we abuse the notation similarly with h to write $h(|S|) = h(S)$. We will first show that if $x > y$ then

$$h(x-1) - h(x) < h(y-1) - h(y) \tag{1}$$

From the definition we have $h(i) = \left(1 - \frac{i}{n}\right)^n$ and using binomial theorem we can write

$$h(i-1) - h(i) = \sum_{j=1}^{n} \left(1 - \frac{i}{n}\right)^{n-j} \left(\frac{1}{n}\right)^j$$

And since this is a decreasing function of i, we have (1). Let $|S \cap S'| = y$ and let $|S| = x$ then $x > y$. Also let $|S \cup S'| - |S| = |S| - |S \cap S'| = z$. We need to show,

$$\begin{aligned}
& g(S \cup S') + g(S \cap S') < g(S) + g(S') \\
\text{or} \quad & g(S \cup S') - g(S) < g(S') - g(S \cap S') \\
\text{or} \quad & g(x+z) - g(x) < g(y+z) - g(y) \\
\text{or} \quad & h(x) - h(x+z) < h(y) - h(y+z) \\
\text{or} \quad & \sum_{i=1}^{z} h(x+i-1) - h(x+i) < \sum_{i=1}^{z} h(y+i-1) - h(y+i)
\end{aligned}$$

And since for each i the term in the summation on the left side is smaller than the corresponding term one on the right, we have strict submodularity of g. □

References

Ben-Porath, E., E. Dekel, and B. L. Lipman (2014). Optimal allocation with costly verification. *The American Economic Review 104*(12), 3779–3813.